

Research article

# MODELING CHROMIUM DEPOSITION ON STEADY STATE WATER TABLE INFLUENCED BY HETEROGENEOUS FORMATION IN SEMICONFINED BED IN OKIRIKA, RIVERS STATE OF NIGERIA.

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## Abstract

Hydraulic conductivity is a reflection of the state of steady water table based on aquifer thickness in a formation. This condition implies that geological setting control the deposition of steady water table in a formation. Hydraulic conductivity expresses water that transmits through a unit cross-sectional area of an aquifer under unit hydraulic gradient. The state of steady water table is through hydraulic conductivity that is commonly called aquifer permeability or coefficient of permeability. This expression reflects the condition of steady water table under normal condition. But the study focuses on heterogeneous formation which has been confirmed to develop an influence in static water table and trace chromium in soil and water environment. Such expression were found to influence the structural setting of water table in the study area, trace metal deposits high degree of concentration under the influence of heterogeneous formation. Based on these factors, mathematical approach was found suitable to mathematically generate solution that will predict the steady water table influenced by heterogeneous formation. Chromium depositions were also found to have deposit higher concentrations based on the heterogeneous structure of the formation. The study is imperative because it has expressed the heterogeneous influence that increased trace chromium in steady state water table in this dimension. Experts in this area will apply this concept developed in the structure of risk assessment and development of quality water including pollution prevention from this source in the study area. **Copyright © WJSTR, all rights reserved.**

**Keywords:** Steady state, Water table, Model prediction, Chromium, and Semiconfined bed.

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## 1. Introduction

The presence of appropriate microorganisms at the actual site of contamination has long been recognized as a key factor in determining if biodegradation will occur, as well as in influencing the rate of biodegradation. A technology for uniform introduction of nutrients and microorganisms has been the principal bottleneck in the successful field implementation of in situ bioremediation (Zappi et al., 1993). An emerging technology for treating petroleum contaminated-sites using in situ bioremediation methods is through the application of direct electric fields to enhance the contact probability of the bacteria and their hydrophobic organic compounds (HOC) by transporting bacteria to contaminant into heterogeneous and low permeability soils for the homogenization of microorganisms in soil (Wick and Harms, 2007) or inverse (Shi and Wick, 2008). Many laboratory (Lahlou et al., 2000), field-scale (Dybas et al., 2002; Major et al., 2002) and modeling studies (Schafer et al., 1998; Shein and Devin, 2002; Kim SB numerical analysis of bacterial, 2006) have been performed to understand and predict microbial transport in porous media. As microbes are generally negatively charged, DC fields will cause their transport towards the anode (DeFlaun and Condee, 1997; Lee, 2001) and/or bacterial migration with the electroosmotic water flow to the cathode (Sun, 2004; Wick et al., 2004). These methods all rely on more selective and gentler ways to move microbes (and contaminants) through the soil and have in common that they do not require the excavation of soil or the mechanical mixing of the soil matrix. The success of using of electric fields depends on the specific conditions encountered in the field, including the types and amount of contaminant present, soil type, pH and organic content (Acar et al., 1996; Wick et al., 2004; Borroni and Rota, 2003).

## **2. Theoretical background**

Steady water table is reflected from the level of geological formation deposited under the influence of formation characteristics. Such geological setting is from the variables that make up the stratification formation. Static water table express lots of variations, this depends on the degree of porosity under the influence of artificial recharge from high rain intensities thus influenced by climatic conditions. Furthermore, the recharge of static water level are determined by the rate of structural stratification of the formation under the influence of disintegration of permeable rocks into grain size that structured in a matrix form generating micropores within the intercedes. The geological setting deposits groundwater at a particular formation or depth depending on the type of strata that can develop good hydraulic conductivity known to be aquiferous zones. Aquifer thickness is determined by the structural sorted grain size from disintegration of the sediments into fine coarse and gravel formations. Aquifers are deposited in consolidated and unconsolidated formation. Some consolidated formations are the fissures or zone of enhanced permeability. These influence specific discharge under distribution of bearing fissures due to the level of the strata micropores that deposit consolidated aquiferous formations. But the focus of this study centred on heterogeneous formation that deposited in lacustrine region where aquifers are not in uniformity. The depositions of chromium were found as a trace metal in the soil and water environment. The heterogeneous influence of the strata develops variation of accumulation of chromium in some regions of the formation. Subject to this relation, it is obvious that at heterogeneous condition of the soil there is a tendency that it will reflect on the concentration of the trace heavy metal in some formation of the soil. Lacustrine deposits that generate heterogeneous formation in the study area

were confirmed to develop high concentration of chromium mostly on where the hydraulic conductivity will experience low percentage. This implies that there is tendency of expressed variation of the soil matrix as deposited in some locations of the study area. To critically analyse in detail the accumulation of the trace metal influencing the steady water table, it is of interest that the environmental conditions are reflected on the deposition of chromium migrating to the static water table. Consequently, it has resulted in experience of trace solute in some aquiferous zones in some part of the study area. Base on these factors the governing equation was formulated to monitor the migration of chromium in steady water tables in the study area.

### 3. Governing equation

$$D \frac{\partial^2 C}{\partial x^2} - \Gamma K \frac{\partial C}{\partial x} = \frac{\partial C(x)}{\partial t} \dots\dots\dots (1)$$

The expression is the governing equation that was formulated to monitor the steady water level influenced by heterogeneous formation and deposition of chromium in soil and water environment. The expressed governing equation provided a platform where all variables found to be influential to the system will be expressed in detail, such expression mathematically is through a derivative function denoted with mathematical symbol,. the governing equation express a mathematical application that will detail the variables function in predicting static water level and heterogeneous formation in soil and water environment.

Boundary condition  $C(o,t) = Co$  for  $t > 0$   $C(z,t) = 0$  and  $C(\infty,t) = 0$  for  $t \geq 0$

The Laplace transform for a function  $f(t)$ , which is defined for all values of  $t \geq 0$  is given as

$$\rho f(t) = \rho s = \int_0^{\infty} e^{-st} f(t) dt \quad (ft) = \rho^{-1} f(s) \dots\dots\dots (2)$$

$$\rho f^1(t) = s\rho f(t) - f(o) \text{ where } f^1(t) = \frac{\partial f}{\partial t} \dots\dots\dots (3)$$

$$\rho f^{11}(t) = s\rho f^1(t) - f^1(o) = s S\rho f(t) - f(o) - f^1(o) = S^2 \rho f(t) - Sf(o) - f^1$$

Taking the Laplace transform of the function c with respect to t, equation (1) changes to

$$D\rho \left[ \frac{\partial^2 C}{\partial x^2} \right] - \Gamma K\rho \left[ \frac{\partial C}{\partial x} \right] = \rho \frac{\partial C(x)}{\partial t} \dots\dots\dots (4)$$

Where  $\rho \left[ \frac{\partial C}{\partial t} \right] = S\rho(c) - C(x,o)$

This implies that  $\bar{C} = \rho(c)$  then  $\rho = \left[ \frac{\partial c}{\partial x} \right] = \frac{\partial}{\partial x} \rho(c) = \frac{\partial \bar{C}}{\partial Z}$  and  $\rho \frac{\partial^2 C}{\partial x^2} = \left[ \frac{\partial^2}{\partial x^2} \right] \rho(c) = \frac{\partial^2 \bar{C}}{\partial x^2}$

Where  $\bar{C}(z) = \rho C(x, t)$ , that is only  $t$  changes to  $s$  and  $x$  is unaffected and  $s$  is the Laplace parameter.

$$\text{At } z = 0; \bar{C}(x) = \int_0^{\infty} e^{-st} C(x, t) dt = \int_0^{\infty} e^{-st} C_o dt = \left. \frac{1}{s} e^{-st} C_o \right|_0^{\infty} = \frac{C_o}{s}$$

$$\text{At } z = \infty; \bar{C}(x) = \int_0^{\infty} e^{-st} C(x, t) dt = 0$$

Therefore at  $z = 0$ ,  $\bar{C}(x) = \frac{C_o}{s}$ , and  $z = \infty$ ,  $\bar{C}(x) = 0$

[This expression is one dimensional flow equation, partial derivative changes to the full derivative;  $s$  is a Laplace parameter, which disappears on taking the inverse].

Transforming the variables into Laplace conditioned the parameters to express their functions are based on the behaviour of steady water table, these are influenced by heterogeneous formation and deposition of chromium. Subject to this condition, it is observed that the variables in the system will be integrated by considering several variations in the strata, the deposition of chromium in heterogeneous formation are influenced by the steady water table under high degree of accumulation. The expressed derived solution were subjected into this transformation to ensure that the parameters are expressed to the condition of the structural stratification of heterogeneous soil including the trace deposition of chromium as it is expressed form equation (2) to the stated equation (4) above.

From the substitution in equation (4) changes to

$$D \left[ \frac{\partial^2 \bar{C}}{\partial x^2} \right] - \Gamma K \frac{d\bar{C}}{dx} = S\bar{C} \dots\dots\dots (5)$$

Let  $\bar{C} = Ae^{\lambda x}$  be the solution of the above linear, second order ordinary differential equation. [This is a standard way of solving this class of equations].

$$\text{Then } \frac{d\bar{C}}{dx} = A\lambda e^{-\lambda x} \text{ and } \frac{d^2 \bar{C}}{dx^2} = A\lambda^2 e^{\lambda x}$$

Substitution of these values in Equation (5) gives

$$DA\lambda^2 e^{\lambda x} - \overline{\Gamma K} A \lambda e^{\lambda x} - S \lambda e^{\lambda x} = 0 \text{ or } \left[ \frac{\overline{\Gamma K}}{D} \lambda - \frac{s}{D} \right] e^{\lambda z} = 0 \dots\dots\dots (6)$$

This will be a solution if the auxiliary equation or the characteristics Equation = 0, that is,

$$\left[ \lambda^2 - \frac{\overline{\Gamma K}}{D} - \frac{s}{D_L} \right] = 0 \dots\dots\dots (7)$$

Static water table in this direction may be found to be influenced by constant concentration and velocity of flow thus despite heterogeneous nature of the soil. Subject to this condition, the derived solution considered this circumstance that may take place in some regions of the formation; these are experienced through the migration of chromium in that direction of flow. This expression is determined by the structural deposition of the formation in some regions despite the heterogeneous nature of the strata in the study location. Therefore, the expression of constant concentration in the derived solution including other parameters in line with the deposited nature of the formation in the study area, thus considered in the derived equation on these process.

Apply a standard quadratic expression and the solution is given as:

$$\lambda = \frac{\frac{\overline{\Gamma K}}{D} \pm \sqrt{\frac{\overline{\Gamma K}^2}{D} + \frac{4s}{D}}}{2}$$

This implies that, either  $\overline{C} = Ae^{\lambda_1 x}$  or  $\overline{C} = Ae^{\lambda_2 x}$  is a solution.  
 However, only the latter satisfies the boundary condition.

At  $z = \infty$ ,  $\overline{C} = \frac{C_o}{s}$ ,  $e^{-\infty} = 0$  {because  $\lambda_2$  is -ve and  $\lambda_1$  is +ve}

Therefore  $\overline{C} = A \left[ e^{\frac{\overline{\Gamma K} - \sqrt{\overline{\Gamma K}^2 + 4sD}}{2D}} \right]^x$  is the function

At  $x = 0$ ,  $\overline{C} = \frac{C_o}{s}$  gives  $A = \frac{C_o}{s}$

Therefore  $\overline{C} = \frac{C_o}{s} \left[ \exp \left[ \overline{k} - \frac{\sqrt{\overline{\Gamma K}^2 + 4sD}}{2D} \right]^x \right]$  is the solution ..... (8)

From Equation (8)  $C(x, t)$  can be determined as  $\rho^{-1} \overline{C}(x)$

Equation (8) can further be written as:

$$\bar{C}(x) = C_o \exp\left(\frac{\overline{\Gamma K}x}{2D}\right) \frac{1}{s} \exp\left[\frac{-x}{\sqrt{D}} \left(\frac{\overline{\Gamma K}^2}{4D} + s\right)^{\frac{1}{2}}\right]$$

Introducing the application of inverse Laplace transform to the above equation gives

$$\begin{aligned} C(x,t) &= \rho^{-1} \bar{C}(x) = \rho^{-1} \left[ C_o \exp\left(\frac{\overline{\Gamma K}x}{2D}\right) \frac{1}{\phi s} \exp\left[\frac{-x}{\sqrt{D}} \left(\frac{\overline{\Gamma K}^2}{D} + s\right)^{\frac{1}{2}}\right] \right] \\ &= \left[ C_o \exp\left(\frac{\overline{\Gamma K}x}{2D}\right) \rho^{-1} \left[ \frac{1}{s} \exp\left[\frac{-x}{\sqrt{D}} \left(\frac{\overline{\Gamma K}}{D} + s\right)\right] \right] \right] \dots\dots\dots (9) \end{aligned}$$

From the Laplace transform table

$$\rho^{-1} \left( \frac{1}{\phi s} \exp\left(-\alpha\sqrt{\beta^2 + s}\right) \right) = \int_0^t \frac{\alpha}{2\sqrt{\pi}\beta} \exp\left[-\left(\frac{\alpha^2}{4u} + \beta^2 u\right) du\right] \dots\dots\dots (10)$$

Where  $\alpha = \frac{x}{\sqrt{D}}$  and  $\beta = \frac{\overline{\Gamma K}}{2\sqrt{D}}$

Therefore,

$$C(x,t) = C_o \exp\left(\frac{\overline{\Gamma K}x}{2D}\right) \left[ e^{-\alpha\beta} \int_0^t \frac{\alpha}{2\sqrt{\pi}\beta} \exp\left[-\frac{\alpha^2}{4u} - \beta^2 u + \alpha\beta du\right] \right] \dots\dots\dots (11)$$

Now only the term in rectangular bracket is considered for further simplification.

$$\text{The term in the bracket} = \left[ e^{-\alpha\beta} \int_0^t \frac{\alpha}{2\sqrt{\pi}\beta} \exp\left[\frac{(\alpha - 2\beta u)^2}{4u} du\right] \right] \dots\dots\dots (12)$$

$$= e^{-\alpha\beta} \int_0^t \left[ \frac{\alpha + 2\beta u}{4\sqrt{\pi u^3}} + \frac{\alpha - 2\beta u}{4\sqrt{\pi u^3}} \right] \exp\left[-\frac{(\alpha - 2\beta u)^2}{4u} du\right] \dots\dots\dots (13)$$

$$= e^{-\alpha\beta} \left[ \int_0^t \frac{\alpha + 2\beta u}{4\sqrt{\pi u^3}} \exp \left[ \frac{(\alpha - 2\beta u)^2}{4u} du \right] + e^{2\alpha\beta} \int_0^t \frac{\alpha - \beta u}{4\sqrt{\pi u^3}} \exp \left[ \frac{(\alpha + 2\beta u)^2}{4\sqrt{\pi u^3}} du \right] \right] \dots\dots (14)$$

Let  $\frac{\alpha - 2\beta u}{\sqrt{4u}} = A$  and  $\frac{\alpha + 2\beta u}{\sqrt{4u}} = B$  .....

Differentiating the term in Equation (15) gives

$$\frac{dA}{du} = \frac{\sqrt{4u}(0 - 2\beta) - 2 \frac{1}{2} \sqrt{u} (\alpha - 2\beta u)}{4u} \quad \text{and} \quad \frac{dB}{du} = \frac{\sqrt{4u}(0 + 2\beta) - 2 \frac{1}{2} \sqrt{u} (\alpha + 2\beta u)}{4u} \dots (16)$$

$$\text{Or } \frac{dA}{du} = \frac{-4\beta\sqrt{u} - \frac{\alpha}{u} + 2\beta\sqrt{u}}{4u} = \frac{-2\beta u - \alpha}{4\sqrt{u^3}} = \frac{(\alpha + 2\beta u)}{4\sqrt{u^3}}$$

$$\text{Or } dA = \frac{-(\alpha + 2\beta u)}{4\sqrt{u^3}} du \quad \text{and} \quad dB = \frac{-(\alpha - 2\beta u)}{4\sqrt{u^3}} du \dots\dots\dots (17)$$

Substituting the term in equation (15) and (17) into equation (11) can be expressed as

$$C(z, t) = C_o \exp \left( \frac{\Gamma K x}{2D} \right) - e^{-\alpha\beta} \left[ \int_{\frac{\alpha - 2\beta t}{\sqrt{4t}}}^{\frac{\alpha - 2\beta t}{\sqrt{4t}}} \exp(-A^2) \frac{dA}{\sqrt{\pi}} - e^{2\alpha\beta} \int_{\frac{\alpha + 2\beta t}{\sqrt{4t}}}^{\frac{\alpha + 2\beta t}{\sqrt{4t}}} \exp(-B^2) \frac{dB}{\sqrt{\pi}} \right] \dots\dots\dots (18)$$

For the limit when  $u = 0$

$$A = \frac{\alpha - 2\beta 0}{0} = \infty, \quad B = \frac{\alpha + 2\beta 0}{0} = \infty, \quad \text{and when}$$

Changing the integral limits in Equation (18), it is given as

$$\frac{1}{2} \frac{2}{\sqrt{\pi}} e^{-\alpha\beta} \int_{\frac{\alpha - 2\beta t}{\sqrt{4t}}}^{\infty} \exp(-A^2) dA + \frac{1}{2} \frac{2}{\sqrt{\pi}} e^{\alpha\beta} \int_{\frac{\alpha + 2\beta t}{\sqrt{4t}}}^{\infty} \exp(-B^2) dB \dots\dots\dots (19)$$

The complimentary error function is defined as  $erfc x = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2) dt$

for which Equation (19) changes to

$$\frac{e^{-\alpha\beta}}{2} \operatorname{erfc} \frac{\alpha - 2\beta t}{\sqrt{4t}} + \frac{e^{\alpha\beta}}{2} \operatorname{erfc} \frac{\alpha + 2\beta t}{\sqrt{4t}} \dots\dots\dots (20)$$

The various combinations of  $\alpha$  and  $\beta$  can be simplified as follows:

$$\alpha\beta = \frac{x}{D} \frac{\overline{\Gamma K}}{2\sqrt{D}} = \frac{\overline{\Gamma K}x}{2K}; \frac{\alpha + 2\beta t}{\sqrt{4t}} = \frac{\frac{x}{\sqrt{D}} + \frac{\overline{\Gamma K}}{\sqrt{D}}}{2\sqrt{t}} = \frac{x + \overline{\Gamma K}t}{2\sqrt{D't}} \text{ and}$$

$$\frac{\alpha - 2\beta t}{\sqrt{4t}} = \frac{\frac{x}{\sqrt{D}} - \frac{\overline{\Gamma K}t}{\sqrt{D}}}{2\sqrt{t}} = \frac{x - \overline{\Gamma K}t}{2\sqrt{D't}}$$

Using these values, equation (20) changes to

$$e \frac{\overline{K}x}{2} \operatorname{erfc} \left[ \frac{x - \overline{\Gamma K}t}{2\sqrt{D}} \right] + e \frac{\overline{\Gamma K}x}{2D} \operatorname{erfc} \left[ \frac{x + \overline{\Gamma K}t}{2\sqrt{D't}} \right]$$

Therefore finally, Equations (12) and (15) changes to

$$C(x,t) = C_o \exp \left( \frac{\overline{\Gamma K}x}{2D} \right) \frac{1}{2} \exp \left( -\frac{\overline{\Gamma K}x}{2D} \right) \operatorname{erfc} \left[ \frac{x - \overline{\Gamma K}t}{2\sqrt{D't}} \right] \frac{1}{2} \exp \left( \frac{\overline{\Gamma K}x}{2D} \right) \operatorname{erfc} \left[ \frac{x - \overline{\Gamma K}t}{2\sqrt{D't}} \right]$$

Or  $C(x,t) = \frac{C_o}{2} \left[ \operatorname{erfc} \left[ \frac{x - \overline{\Gamma K}t}{2\sqrt{D't}} \right] + \exp \left( \frac{\overline{\Gamma K}x}{D} \right) \operatorname{erfc} \left[ \frac{x + \overline{\Gamma K}t}{2\sqrt{D't}} \right] \right] \dots\dots\dots (21)$

The expression in [21] is the final derived model equation; the system was developed to monitor steady water level and the deposition of chromium in the study location. Several factors were considered in the system that should influence heterogeneous formation and deposition of trace metal in soil and water environment. The derived solution expressed several conditions based on the stratification of the formation to ensure that static water level is predicted in the study area. Heterogeneous formation was found to develop a platform that trace chromium generate variation in concentration. These factors were considered by the governing equation derived with this mathematical method, this is to generate the final expressed model that will predict steady state water table under the influence of heterogeneous formation and trace heavy metal (chromium).

#### 4. Conclusion

Steady state water table is known to deposit based on geological setting in any soil formation; although, variations may be found in seasons under the influence of high rain intensity in areas where we have such climatic condition influence. Furthermore, it is observed that groundwater under natural condition flows from area of recharge



normally from aquiferous zone, outcrop area to point of discharge either as spring, river or in the sea. The driving force of this groundwater flow is the hydraulic head are with difference level. Groundwater surface are between the recharge and discharge areas. The flow of water through the saturated zone of an aquifer is represented by a mathematical equation. This aquifer storativity has facet unconfined aquifer storage or specific yield. The yield of an aquifer is the volume of water which will drain from a unit volume of aquifer under gravity alone. Subject to steady water table, it is expressed based on these factors, which guide the principle of its deposition in the study location. The heterogeneous condition of the formation may develop influence based on the formation characteristics that may express slight variables depending on the structure of the soil matrix.

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